ON CONDITIONS ON THE BOW SHOCK IN VISCOUS GAS FLOW PAST A BLUNT BODY

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Conditions in the bow shock originating in hypersonic viscous gas flow past a blunt body at low Reynolds numbers, which are needed to determine the influence of the velocity of displacement of the boundary layer on the external flow, are here considered in a more general formulation than in [1]. Conditions on the shock are found by the method of internal and external expansions, from which the conditions customarily used on a shock, considered as a mathematical surface, and the conditions obtained in [1] result as particular cases. These conditions are compared, and the reasons impelling the author to consider the conditions on the shock obtained in [1] are clarified (*).

1. Just as in [1], let us consider the plane or axisymmetric problem of uniform hypersonic perfect viscous gas flow around a contour (Fig.1). Here AOA' is the contour of the streamlined body; the domain 4 is the boundary layer; the domain 2 is a shock considered as a domain with large gradients of the gas parameters.

It is assumed that an arc of the contour (from the point O to the limiting characteristics of the inviscid flow) is an analytic curve; the gas is perfect, i.e., its equation of state is $p=R\rho T$, where p is the pressure, ρ the density, T the absolute temperature, R the gas constant; the specific heat at constant pressure c_p and constant volume c_v are constants; the internal energy is $e=c_v T$; the viscosity coefficients μ and λ are functions only of T; the Prandtl number σ is constant. The gas flow is described by the Navier-Stokes equations and its flow is laminar.

We denote the unperturbed flow parameters with the subscript $\sim$ thus: M_∞ is the unperturbed flow Mach number, U_∞ is its velocity. If $\mu,\lambda\to 0$ (σ=const), then the thickness of the domain 2 approaches zero and the domain 2 approximates some surface CBC' (the surface of the shock in the inviscid problem) without limit.

Let us introduce an s,n curvilinear coordinate system (Fig.1). Here s and n are measured along the arc BC and along its normal. Then if the linear quan-

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*) The author committed an oversight in [1]. The method of characterstics in the form proposed in [1] is incorrect. In this connection, the author's statement on the uniqueness of the solution of the problem in the supersonic part of the flow around a blunt body, with the boundary conditions on the bow wave proposed in [1], is false. I. N. Mursinov mentioned these facts to the author.

tities are referred to the radius of curvature a of the body at the point O, the gas velocity to U_∞, the pressure to $\rho_\infty U_\infty^2$, the density to ρ_∞ the temperature to $U_\infty^2 c_p^{-1}$, the entropy and enthalpy of the gas to c_p and U_∞^2, respectively, the viscosity coefficients to the value of μ at $T = U_\infty^2 c_p^{-1}$, , then the continuity, momentum, energy equations and the equation of state of the gas in the chosen coordinate system, are written in the form presented in [2 and 1]. Let us consider [1] to be known. The parameter, characterizing the effect of the viscous force, is

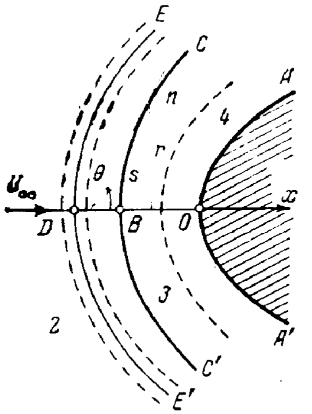

Fig.1

$$\varepsilon = \left[\frac{\mu(U_\infty^2 c_p^{-1})}{\rho_\infty U_\infty a}\right]^{1/2} \qquad \left(\begin{matrix}\mu = \text{const}\cdot \\ \varepsilon = R_\infty^{-1/2}\end{matrix}\right)$$

Here R_∞ is the Reynolds number of the unperturbed flow.

2. The solution of the Navier-Stokes equations in domain (3) will be represented by asymptotic expansions [2] of the form

$$f = F_0(s, n) + \varepsilon F_1(s, n) + \ldots \tag{2.1}$$

where f is understood to stand for the quantities p, ρ, u, v, T; here u, v are the velocity components in the direction of increasing s and n, respectively; $F_0(s,n)$ are parameters of the solution of the inviscid problem; the terms $F_1(s,n)$ characterize the influence of the "velocity of displacement" of the boundary layer.

To find the terms $F_1(s,n)$, it is necessary to obtain conditions on the shock ($n=0$) for them. The method of "internal and external" expansions was utilized to solve this problem in [1]. Expansions of the form (2.1) were taken for u, v, p, ρ, T in domains 1 and 3 in Fig.1.; these are the "external" solution. In contrast to [1], let us assume that although the shock "thickness", the domain 2, is a quantity $O(\varepsilon^2)$, the domain 2 with respect to CBC' is however shifted by a quantity $O(\varepsilon)$. If EDE' denotes some line lying within the domain 2, its equation may be written as

$$n = \varepsilon\varphi(s,\varepsilon) = \varepsilon\varphi_0(s) + \ldots \tag{2.2}$$

We take an expansion of the type

$$f = f_0(s, N) + \varepsilon f_1(s, N) + \ldots, N = [n - \varepsilon\varphi(s, \varepsilon)]\,\varepsilon^{-2} \tag{2.3}$$

within the shock (domain 2 in Fig.1).

In [1] we had $N = n\varepsilon^{-2}$. This is the "internal" expansion.

Let us assume that $F_0(s,n)$, $F_1(s,n)$, ... in domains 1,3 Fig.1 are represented by asymptotic power series in n as $n \to 0$, then from (2.1) for small n we obtain

$$f = [F_{00}(s) + F_{01}(s)\,n + \ldots] + \varepsilon\,[F_{10}(s) + F_{11}(s)\,n + \ldots] + \ldots \tag{2.4}$$

After passage from n to N and regrouping terms of the series (2.4), we have

$$f = [F_{00}(s)] + \varepsilon\,[F_{10}(s) + F_{01}(s)\,\varphi_0(s)] + \ldots \tag{2.5}$$

The expansion (2.5) should represent f for large N and small n (the condition of "matching" of the "internal" and "external" expansions.

It follows from (2.5) that the "internal" expansion should have the form (2.3) and

$$f_0 \to F_{00}^+(s), \qquad f_1 \to F_{10}^+(s) + F_{01}^+(s)\,\varphi_0(s), \qquad N \to +\infty \tag{2.6}$$

$$f_0 \to F_{00}^-(s), \qquad f_1 \to F_{10}^-(s) + F_{01}^-(s)\varphi_0(s), \qquad N \to -\infty \tag{2.6 cont.}$$

$$\left(\frac{\partial f_0}{\partial N}, \frac{\partial f_1}{\partial N} \to 0, \quad N \to \pm\infty\right)$$

3. The subsequent procedure is no different from that in [1]. The Navier-Stokes equations are transformed to the N and s variables, taking into account that

$$\left.\frac{\partial f}{\partial s}\right|_n = \left.\frac{\partial f}{\partial s}\right|_N - \frac{1}{\varepsilon}\varphi' \frac{\partial f}{\partial N}\left(\varphi' = \frac{d\varphi}{ds}\right) \tag{3.1}$$

wherein the expansions (2.3) are substituted; systems of equations for the $f_0(s,N)$ and $f_1(s,N)$ are obtained by equating coefficients of identical powers of ε. If we use notation $\{f\} = (f)_{N\to+\infty} - (f)_{N\to-\infty}$, then exactly as in [1], by taking account of (2.6), we obtain from these systems of equations

$$\{\rho_0 v_0\} = 0, \quad \{u_0\} = 0, \quad \{\rho_0 v_0^2 + p_0\} = 0, \qquad \left\{\frac{v_0^2}{2} + \frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0}\right\} = 0, \qquad \gamma = \frac{c_p}{c_v}$$

$$\{\rho_0 v_1 + \rho_1 v_0 - \varphi_0' u_0 \rho_0\} = 0, \quad \{u_1 + v_0\varphi_0'\} = 0 \left\{\frac{\gamma}{\gamma-1}\left(\frac{p_1}{\rho_0} - \frac{p_0}{\rho_0^2}\rho_1\right) + v_0(v_1 - u_0\varphi_0')\right\} = 0$$

$$\{2\rho_0 v_0 v_1 + \rho_1 v_0^2 + p_1\} = 0, \qquad \varphi_0' = d\varphi_0 / ds \tag{3.2}$$

Here according to (2.6)

$$f_0 = F_{00}(s) = (F_{00})_{n=\pm 0}, \quad f_1 = F_{10}(s) + F_{01}(s)\varphi_0(s) = \left(F_1 + \frac{\partial F_0}{\partial n}\varphi_0\right)_{n=\pm 0}$$

Here $f = p, \rho, u, v$ $\{f\} = (f)_{n=+0} - (f)_{n=-0}$. The relations (3.1) are the customary conditions on a strong discontinuity in the inviscid problem. The relations (3.2) differ from those in [1] by the additional terms containing φ_0 and by the fact that $(f_1)_{N\to\pm\infty} = (F_1)_{n=\pm 0}$ are replaced

$$(f_1)_{N\to\pm\infty} = \left(F_1 + \frac{\partial F_0}{\partial n}\varphi_0\right)_{n=\pm 0}$$

Exactly as in [1], Equations (3.2) agree with those conditions which are obtained if it is assumed that the shock is a mathematical surface $n = \varepsilon\varphi_0(s)$.

4. The flow in domain 1 of Fig.1 is not known to accuracy $O(\varepsilon)$ in advance. The conditions for perturbation, damping as $x \to -\infty$ (see [1]) remain arbitrary, u_1, v_1 as $N \to -\infty$ $(n = -0)$, and p_1, ρ_1 are connected by means of the relations

$$\frac{p_1}{p_0} = \gamma\frac{\rho_1}{\rho_0}, \qquad p_1 = -(v_1 \sin\theta + u_1 \cos\theta) \tag{4.1}$$

The relations (3.1), (3.2) and (4.1) on the shock have been obtained under very general assumptions. To determine p_1, ρ_1, u_1, v_1 uniquely from them it is necessary to know u_1, v_1 for $n = -0$ and φ_0 (the parameters of the inviscid problem are considered known). Apparently these relations do not permit a unique determination of the quantities with subscript 1 in the transonic domain [1], hence, it is necessary to make additional assumptions. Here are the following fundamental possibilities:

Version 1. We assume that $u_1 = v_1 = 0$ for $n = -0$; from (4.1) it follows

$$p_1 = \rho_1 = T_1 = 0 \quad \text{for} \quad n = -0$$

Version 2. We assume that $\varphi_0 \equiv 0$

Version 3. We assume that at $n = -0$

$$u_1 = F_1(\varphi_0, \varphi_0', \ldots), \quad v_1 = F_2(\varphi_0, \varphi_0', \ldots)$$

where F_1, F_2 are some functions of their arguments.

The first version is the most attractive. Here, as a consequence of $u_1 = v_1 = p_1 = \rho_1 = T_1 = 0$ for $n = -0$, we obtain that there are no $O(\epsilon)$ perturbations ahead of the shock, the conditions on the shock agree with the conditions obtained by the perturbation method from the conditions on the shock in the inviscid problem. And the whole problem of determining u_1, v_1, p, ρ_1, T_1 agrees with the problem of determining the inviscid flow perturbations around a blunt body, whose surface has been changed slightly, if the method of formal expansion in powers of ϵ is used.

However, the example presented in the Vaglio-Laurin paper [3] (a profile in transonic flow) and his investigation of the problem of flow past a blunt body by the integral relations method, show that linear systems, obtained by formal expansions in powers of a small parameter, to determine perturbations in the case of transonic problems, may possess solutions whose derivatives become infinite on some lines in the transonic domain.

In order to obtain a smoother solution, the boundary conditions may be weakened. For example, if the Tricomi problem is solved for the Tricomi equation, but we specify the desired function on the second characteristic, then a solution of the problem is possible, but in a class of functions having discontinuities of the first derivatives on the parabolic line. To eliminate this discontinuity, it is required that the boundary condition be eliminated from the second characteristic.

Being guided by similar considerations, the author considered the second version in [1] where one condition less is obtained on the shock than in the first version, and there are $O(\epsilon)$ perturbations ahead of the shock. The question of the existence and uniqueness of the transonic problem, with conditions of the second version on the shock, remained open. An indirect argument in favor of the second version in [1] was the correctness of the computations by the metnod of characteristics in the supersonic part of the flow.

However, an investigation for a wedge (at some distance from the vertex), where the solution had successfully been obtained in analytic form, showed that the conditions on the shock in the second version are not sufficient for uniqueness of the solution in the supersonic part of the flow. It hence follows that the method of characteristics, in the form proposed in [1], is not correct.

In this connection, it may be assumed that it is necessary to utilize the conditions on the shock in the second version in the transonic domain, and the conditions of the first version after the limiting characteristics. But such a construction, although possible, seems artificial to the author, and therefore, he does not use the conditions of the second version. The correct solution of the problem will evidently be the following.

Conditions on the shock in the first version should be utilized. If a singular line occurs in the transonic domain, it will be necessary to introduce a supplementary intermediate expansion in ϵ in its neighborhood. It is also necessary to elucidate how valid is the assumption $u_1 = v_1 = 0$ $(p_1 = \rho_1 = T_1 = 0)$ at $n = -0$. The assumption of the third version, in which there are $O(\epsilon)$ perturbations ahead of the shock, is more general.

In conclusion, the author would like to turn attention to the following. Nonlinear inviscid gas, equations are often utilized to find the gas flow perturbations because of the velocity of displacement, but the body contour is altered correspondingly. Evidently linear equations for the perturbations should be utilized since, as the examples in [3] show, the results of solving nonlinear and linear systems for the perturbations may differ substantially for the transonic problems.

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